

Suspension of Convertibility versus Deposit Insurance: A Welfare Comparison

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Abstract. This paper introduces risk-averse preferences in Chari and Jagannathan (1988). A first motivation for this extension is to give a positive role for a financial intermediary in the economy, who offers risk-sharing contracts to liquidity seeking individuals. In this framework, both information-induced and pure panic runs will occur. The second motivation is to complete Chari and Jagannathan's welfare analysis by comparing suspension of convertibility and deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation). It is shown that the choice between the two contracts depends on the level of risk aversion, the intertemporal discount factor and the attributes about the underlying technology.

Key words: bank runs, deposit contracts, deposit insurance, optimal risk sharing, suspension of convertibility.

JEL classification codes: G21, G28.

1. Introduction

The banking system has traditionally been vulnerable to the problem of bank runs, in which many or all depositors at a bank attempt to withdraw their funds simultaneously. If these withdrawals at a particular bank then spread across banks in the same region or country they may generate a banking panic. These financial crises are costly and are prevented with different intervention measures. One example of such crises is the Great Depression (1929–1933) which had a significant impact on the banking system of the US. From 1930 to 1933 the number of bank failures in the US averaged over 2000 per year.¹ Furthermore, many emerging countries have had severe problems in their banking systems in recent years.

* This research is partially funded by the Spanish Ministry of Education and Culture, projects: PB97-0089, DGES and SEC 2001-1169. The author owes special thanks to Sudipto Bhattacharya, Sandro Brusco, Elena Carletti, Steve Smith and an anonymous referee for their very helpful comments in earlier versions of this paper.

¹ See Mishkin (1995).

Given the historical importance of panics and their current relevance, it is important to understand why they occur and what policies should be implemented to deal with them.

In this sense, the theoretical research on banking has focused on understanding the economic role of banks in the financial system and why they are left vulnerable to bank runs (e.g., Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), Alonso (1996) and Allen and Gale (1998) among others). This stream of research has approached banking panics through two different types of models.

First, the models of pure panic runs, in which bank runs occur as random phenomena, with no correlation with other economic variables. Diamond and Dybvig (1983), following Bryant (1980), attempted to formalize the notion of liquidity seeking behaviour by individuals and its implications for the design of their financing contracts. They demonstrated that demand deposit contracts, which convert highly illiquid assets into liquid deposits, provide a rationale both for the existence of banks and for their vulnerability to runs. The model analyzes two different intervention policies to cope with bank runs. They advocate that if there is no aggregate uncertainty, a suspension of convertibility policy would eliminate the bank run equilibrium. Otherwise, a deposit insurance policy would be more effective.

Diamond and Dybvig's model has attracted several criticisms. Gorton (1988), in an empirical study of bank runs in the US during the National Banking Era (1863–1913), found support for the notion that bank runs tended to occur after business cycle peaks, and so they are not the result of “sunspots”, as Diamond and Dybvig's model suggests. In a latter paper, Bhattacharya and Thakor (1993) question the optimality of deposit insurance in Diamond and Dybvig's model. Their claim is that both suspension of convertibility and deposit insurance are second best measures. Moreover, Engineer (1989) shows that if the horizon in Diamond and Dybvig is extended to four periods and one adds a subset of agents with preferences weighted toward consumption in the fourth period, then suspension of convertibility does not always prevent a run.

Second, models of information-induced runs assert that bank runs occur due to the diffusion of negative information among depositors about bank's solvency (e.g., Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Alonso (1996) and Allen and Gale (1998)). Chari and Jagannathan (1988) drew on both information-induced and pure panic runs models² to study the effects of extra market constraints, such as suspension of convertibility on bank runs. They concluded that such constraints prevent bank runs and result in superior allocations. Despite the importance of this contribution, it raised considerable criticisms due

² In the model, panic runs may occur due to the fact that uninformed individuals condition their beliefs about the bank's long-term technology on the size of the withdrawal queue at the bank. If this size is large (due to a high liquidity shock only) they may nevertheless infer sufficiently adverse information to precipitate a bank run.

to the ambiguous role of banks or any other financial intermediary in the model, as individuals were risk neutral. Jacklin and Bhattacharya (1988) also model information-induced runs that occur as a result of a negative signal received by some depositors, regarding the future value of the bank's investment. However, the authors are concerned with examining the relative degrees of risk-sharing provided by bank deposit and traded equity contracts. They focused on the relationship between riskiness and information on the stream of returns and the welfare properties of deposit versus equity contracts. They found that deposit contracts tended to be better for financing low risk assets. Alonso (1996) demonstrates using numerical examples, that in the Jacklin and Bhattacharya framework contracts where runs occur may be better than contracts that ensure runs do not occur because the former improve risk sharing. An interesting addition to this literature is the recent paper by Allen and Gale (1998). In contrast to previous literature,³ which has focused mainly on modelling bank runs, this paper analyzes the optimal intervention policy that should be implemented, if any, to deal with panics. The paper shows that under certain circumstances, bank runs can be first best efficient, as they allow efficient risk sharing among depositors and they allow banks to hold efficient portfolios. This result seems to contradict the traditional history of regulation, based on the premise that banking panics are bad and should be eliminated. However, if there are liquidation costs or markets for risky assets are introduced, *laissez-faire* is no longer optimal, and central bank intervention is needed to achieve the first best.

This paper presents a model consistent with the business cycle view of the origins of banking panics. As in Chari and Jagannathan (1988), there is a signal extraction problem, where part of the population observes a signal about future returns. Others must then try to deduce from observed withdrawals whether a non favorable signal was received by this group or whether liquidity needs happen to be high. Panics will occur not only when the outlook is poor but also when liquidity needs turn out to be high. However, the key difference of the present model with respect to Chari and Jagannathan (1988) is first, individuals are risk-averse and the demand deposit contract is presented. The importance of this extension rests on the distinctive role of financial intermediaries in the economy, especially by providing insurance to individuals subject to preference shocks. Second, this paper provides an adequate framework to compare between the two second best measures to deal with runs, namely suspension of convertibility or deposit insurance. The choice between the two measures is examined, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation, see Bhattacharya, Boot and Thakor (1998) for a discussion of these costs). Numerical simulations show that the choice between suspension of convertibility or deposit insurance may depend on exogenous parameters such as the level of risk aversion, the agents' intertemporal discount factor and the attributes about the long-term asset return.

³ Another exception is Bhattacharya and Gale (1987) or a few other references mentioned in Allen and Gale (1998).

The structure of the paper is as follows: The basic framework of the model is presented in section 2. Section 3 characterizes the demand deposit contract that allows for the possibility of runs and describes two different measures that have traditionally been used by banks in order to prevent runs. A welfare comparison of both measures, using numerical examples, is presented in Section 4. Section 5 provides a discussion of the main assumptions considered in the paper and finally the concluding remarks are summarized in section 6.

2. The model

There is a three period economy ($T = 0, 1, 2$) and one single commodity. On the consumer side of the economy, there is a continuum of ex ante identical agents that are endowed with one unit of this consumption good at $T = 0$. At $T = 1$, they are subject to a privately observed preference shock and they can be of either of two types. The difference between types is that type 1 agents derive relatively more utility for consumption in period one with respect to type 2 agents. A random fraction \hat{t} of individuals are of type 1. This random variable can take a value t_1 with probability r or t_2 with probability $1 - r$ (and $t_1 < t_2$). In order to present a parameterized example of the model, the following form for the utility function is assumed:

$$U^i(c_1, c_2, \rho_i) = \rho_i \frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1 - \rho_i) \frac{(k+c_2)^{1-\gamma}}{1-\gamma} \quad \gamma > 1, k > 0 \quad (1)$$

$i = 1, 2$

where k is a positive constant, $0 < \rho_2 < \rho_1 < 1$ and $\rho_1 = 1 - \rho_2 = \rho > 0.5$.⁴

At $T = 1$ a random fraction, $\tilde{\alpha}$, of type 2 depositors receives a perfectly informative signal on the future value of the bank's assets. This random variable $\tilde{\alpha}$ can take a value α with probability q and 0 with probability $1 - q$.

On the side of the intermediary, there is one investment technology that for each unit invested at $T = 0$ generates a random return \tilde{R} at $T = 2$. The production process is infinitely divisible. Any portion of the production process can be interrupted at $T = 1$ and yield a total return equal to the initial investment.⁵ The value of the random return will be $0 \leq R_l < 1$ with probability p and $R_h > 1$ with probability $1 - p$. It is also assumed that $\tilde{R} = pR_l + (1 - p)R_h > 1$ and that the probability (p) of the low return occurring is sufficiently small. A motivation for this assumption is given in Section 5.

Finally, and following Chari and Jagannathan (1988), the following parameter restriction is assumed:

$$t_2 = t_1 + \alpha(1 - t_1). \quad (2)$$

⁴ The positive constant k keeps the objective function bounded for all weakly positive consumption levels.

⁵ See Diamond and Dybvig (1983) for a motivation for this assumption.

Table I. States of nature

θ_i i	State $\tilde{t} \tilde{\alpha} \tilde{R}$	Prob.	Aggregate demand for liquidity at $T = 1$ ($\tilde{C}T_1$)	$\tilde{C}T_1$ (Theorem 1 satisfied)
1	$t_1 0 \tilde{R}$	$r(1 - q)$	$t_1 c_{11} + (1 - t_1)x_{1U}$	$t_1 c_{11} + (1 - t_1)c_{12}$
2	$t_1 \alpha R_h$	$r(1 - p)q$	$t_1 c_{11} + (1 - t_1)[\alpha c_{12} + (1 - \alpha)x_{1U}]$	$t_1 c_{11} + (1 - t_1)c_{12}$
3	$t_1 \alpha R_l$	$r p q$	$t_1 c_{11} + (1 - t_1)[\alpha c_{11} + (1 - \alpha)x_{1U}]$	c_{11}
4	$t_2 0 \tilde{R}$	$(1 - r)(1 - q)$	$t_2 c_{11} + (1 - t_2)x_{1U}$	c_{11}
5	$t_2 \alpha R_h$	$(1 - r)(1 - p)q$	$t_2 c_{11} + (1 - t_2)[\alpha c_{12} + (1 - \alpha)x_{1U}]$	$t_2 c_{11} + (1 - t_2)c_{12}$
6	$t_2 \alpha R_l$	$(1 - r)pq$	$t_2 c_{11} + (1 - t_2)[\alpha c_{11} + (1 - \alpha)x_{1U}]$	c_{11}

This assumption allows for some confusion in the observation of the signal by uninformed agents.

The three random variables are assumed to be independently distributed. Together, they describe the state of nature, $\tilde{\theta} = (\tilde{t}, \tilde{\alpha}, \tilde{R})$. Table I (columns 2 and 3) shows the different states of nature and their associated probabilities.⁶

The time structure of the model is summarized as follows: At $T = 0$ individuals deposit their endowment in the bank, that has a comparative advantage in investing in the risky asset. In exchange for their endowment depositors are offered a menu of contracts. These contracts provide consumers with flexibility in the timing of consumption. It is assumed (in line with the standard banking literature) that the banking sector is perfectly competitive, so banks offer risk sharing contracts that maximize depositors' ex ante expected utility, subject to a zero profit constraint. At $T = 1$, when the preference and information shocks are realized, individuals will decide on the amount they wish to withdraw at $T = 1$ and $T = 2$. In this context, both information-induced and pure panic runs will occur. Two traditional policies to cope with runs are analyzed. It is shown that in some cases, suspension of convertibility may be the preferred measure to prevent runs, while in other circumstances deposit insurance turns out to be a better policy.

3. The demand deposit contract

As mentioned before, banks offer risk sharing contracts to individuals that are uncertain about their liquidity needs. These contracts allow individuals to adjust their pattern of withdrawals to their consumption needs. Formally, a demand deposit contract is defined as a contract that requires an initial investment at $T = 0$ with the intermediary in exchange for the right to withdraw per unit of initial investment (at the discretion of depositor and conditional on the bank's solvency) either c_{11} units

⁶ It should be pointed out that although there are 8 possible states of nature, they have been simplified to 6 states, due to the fact that in states 1 and 4 there is no information available in the economy about the realization of \tilde{R} . This allows to model pure panic runs.

in period 1 and \tilde{c}_{21} units in period 2 or c_{12} units in period 1 and \tilde{c}_{22} units in period 2. As shown by Jacklin (1987), the demand deposit contract optimally combines the two types of deposits that banks usually hold, a time deposit and a more typical demand deposit contract.

That is, at $T = 0$, individuals deposit their unit of endowment at the bank and are offered a menu of contracts, where they receive a fixed payment at date 1 and a random one at date 2. The second period random payment will depend on the withdrawal queue size \tilde{t} and the random return \tilde{R} . This uncertain second period return reflects the fact that having invested in a risky asset the bank may not be able to make its promised payments at date 2. One way to think of this is that the bank promises an amount it will be able to pay if $R = R_h$. If $R = R_l$ the bank is considered insolvent and depositors get R_l/R_h of their promised payment.⁷ The optimal contract choice for a deposit contract, in the absence of interim information,⁸ can then be obtained as a solution to the following problem:

$$\max_{c_{1i}, c_{2j}^{t_i}, \tilde{K}^{t_i}} \left\{ E_{\tilde{R}, \tilde{t}} \left[\tilde{t} U^1(c_{11}, \tilde{c}_{21}, \rho_1) + (1 - \tilde{t}) U^2(c_{12}, \tilde{c}_{22}, \rho_2) \right] \right\}, \quad (3)$$

$$\begin{aligned} \text{s.t. } & t_1 c_{11} + (1 - t_1) c_{12} \leq K^{t_1} \\ & t_1 c_{21}^{t_1} + (1 - t_1) c_{22}^{t_1} \leq (1 - K^{t_1}) R_h \\ & t_2 c_{11} + (1 - t_2) c_{12} \leq K^{t_2} \\ & t_2 c_{21}^{t_2} + (1 - t_2) c_{22}^{t_2} \leq (1 - K^{t_2}) R_h, \end{aligned} \quad (4)$$

$$E_{\tilde{R}, \tilde{t}} \left[U^j(c_{1j}, \tilde{c}_{2j}, \rho_j) \right] \geq E_{\tilde{R}, \tilde{t}} \left[U^j(c_{1i}, \tilde{c}_{2i}, \rho_j) \right] \text{ for } i \neq j; i, j = 1, 2, \quad (5)$$

where c_{1i} represents consumption at $T = 1$ for the type i agent, $c_{2i}^{t_1}, c_{2i}^{t_2}$ consumption at $T = 2$ for the type i agent conditional on the two possible realizations of the withdrawal queue size t_1 and t_2 respectively. If $R = R_h$ the bank pays its promised payment $c_{2i}^{t_1}$ or $c_{2i}^{t_2}$. However, if $R = R_l$ the bank is considered insolvent in the second period and pays R_l/R_h of its promised payments. K^{t_1} and K^{t_2} are the proportions of the long term investment to be liquidated at $T = 1$ conditional on the realization of \tilde{t} . The first four constraints represent resource balance constraints while the last two show the incentive compatibility constraints that are expressed in expected terms as the contract ignores the information shock at date 1.

Applying Kunh–Tucker conditions, the optimal solution is obtained (a detailed calculation of this problem is given in Appendix A).

⁷ Jacklin and Bhattacharya (1988) have shown that this policy is socially optimal given the assumed preference structure.

⁸ See Section 5 for a discussion of this assumption.

3.1. SUSPENSION OF CONVERTIBILITY

As mentioned before, individuals deposit their endowment in the bank and in exchange are offered the demand deposit contract described in the previous section. At $T = 1$, conditional on the preference and information shocks, depositors will decide on the amount they wish to withdraw at each date. In making their withdrawal decision, agents take into account the fact that whenever runs occur the bank can suspend convertibility at the level of the highest proportion of type 1 consumers (t_2).

In the following, the decisions of the different types of agents will be described. It will be assumed that type 1 and informed type 2 individuals make their withdrawal decision first, followed by the rest of uninformed type 2 individuals. This last group of agents will try to infer the state of nature by looking at the size of the “first stage” queue, that is, the amount that is being withdrawn by type 1 and informed type 2 agents.

First, the withdrawal decision of type 1 consumers is trivial. As these agents face liquidity needs at date 1, they will always select their own contract or withdrawal stream (c_{11}, \tilde{c}_{21}) for $\theta = 1$ to 6.

Second, informed type 2 individuals condition their withdrawal decision on the perfectly informative signal they receive. Whenever the signal indicates that the outcome will be low the next date, they will choose to withdraw their funds from the bank, that is, to select the type 1 contract, provided the following condition holds:

$$\begin{aligned} \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left[r \frac{(k+c_{21}^t \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} + (1-r) \frac{(k+c_{21}^{t_2} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \right] \geq \\ \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left[r \frac{(k+c_{22}^t \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} + (1-r) \frac{(k+c_{22}^{t_2} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \right]. \end{aligned} \quad (6)$$

The left side of Equation (6) represents the expected utility that an informed type 2 obtains by misrepresenting in a bad state and the right side of (6) is the expected utility of not misrepresenting in a bad state.

Similarly, the optimal decision of informed agents is to maintain their funds in the bank if they receive a positive signal, provided the following condition holds:

$$\begin{aligned} \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left[r \frac{(k+c_{21}^{t_1})^{1-\gamma}}{1-\gamma} + (1-r) \frac{(k+c_{21}^{t_2})^{1-\gamma}}{1-\gamma} \right] \leq \\ \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left[r \frac{(k+c_{22}^{t_1})^{1-\gamma}}{1-\gamma} + (1-r) \frac{(k+c_{22}^{t_2})^{1-\gamma}}{1-\gamma} \right]. \end{aligned} \quad (7)$$

The demand for liquidity of informed agents, conditional on each state of nature, will be denoted by \tilde{x}_{1I} , \tilde{x}_{2I} and is summarized in the following lemma.

LEMMA 1. Assume Equations (6) and (7) are satisfied, then the optimal decision of informed individuals is: $\tilde{x}_{1I} = c_{11}$, $\tilde{x}_{2I} = \tilde{c}_{21}$ in states $\theta = 3$ and 6 and $\tilde{x}_{1I} = c_{12}$, $\tilde{x}_{2I} = \tilde{c}_{22}$ in states $\theta = 2$ and 5.⁹

Finally, uninformed type 2 agents condition their withdrawal decision on the amount that is being withdrawn at $T = 1$ by type 1s and informed type 2s. They will try to infer the state of nature by “the size of the queue”. However, this size can be large enough due to both a negative information shock or to a liquidity shock (those states of nature in which the highest proportion of type 1 agents is realized, i.e. $t = t_2$). As a result of this noisy signal, uninformed depositors may sometimes erroneously withdraw their funds from the bank. Their demand for liquidity will be represented by \tilde{x}_{1U} , \tilde{x}_{2U} .

In order to characterize the optimal withdrawal decision for uninformed depositors, let $\tilde{C}T_1$ represent the level of aggregate demand for liquidity at $T = 1$ for each state of nature, that is,

$$\tilde{C}T_1 = \tilde{t}c_{11} + (1 - \tilde{t})[\tilde{\alpha}\tilde{x}_{1I} + (1 - \tilde{\alpha})\tilde{x}_{1U}]. \quad (8)$$

The values of $\tilde{C}T_1$ are shown in Table I (column 4). It can be observed that for any choice of uninformed depositors there is always confusion between states 3 and 4, as $t_2 = t_1 + \alpha(1 - t_1)$.

Let us assume that the information partitions of uninformed type 2 depositors in the conjectured equilibrium are:

- (a) $\tilde{C}T_1 = t_1c_{11} + (1 - t_1)c_{12}$ which implies states $\theta = 1, 2$.
- (b) $\tilde{C}T_1 = c_{11}$ which implies states $\theta = 3, 4$ and 6.
- (c) $\tilde{C}T_1 = t_2c_{11} + (1 - t_2)c_{12}$ which implies state $\theta = 5$.

Given the above information partitions, it can be shown that the optimal withdrawal decision for the uninformed agents is $\tilde{x}_{1U} = c_{12}$ and $\tilde{x}_{2U} = \tilde{c}_{22}$ in states $\theta = 2$ and 5, provided the following conditions hold:

$$\begin{aligned} \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \left[\frac{(k+c_{21}^{t_1})^{1-\gamma}}{1-\gamma} \pi_{1(1,2)} + \frac{\left(k+c_{21}^{t_1} \frac{R_l}{R_h}\right)^{1-\gamma}}{1-\gamma} \pi_{2(1,2)} \right] &\leq \\ \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \left[\frac{(k+c_{22}^{t_1})^{1-\gamma}}{1-\gamma} \pi_{1(1,2)} + \frac{\left(k+c_{22}^{t_1} \frac{R_l}{R_h}\right)^{1-\gamma}}{1-\gamma} \pi_{2(1,2)} \right], & \end{aligned} \quad (9)$$

$$\rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \frac{(k+c_{21}^{t_2})^{1-\gamma}}{1-\gamma} \leq \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \frac{(k+c_{22}^{t_2})^{1-\gamma}}{1-\gamma} \quad (10)$$

⁹ Obviously, in states $\theta = 1$ and 4 no withdrawal decision is undertaken by informed depositors, as all agents are uninformed ($\alpha = 0$).

$$\text{and } \pi_{1(1,2)} = \frac{(1-p)r}{r[(1-q)+(1-p)q]}, \pi_{2(1,2)} = \frac{pr(1-q)}{r[(1-q)+(1-p)q]}.$$

The left side of Equation (9) is the expected utility that the uninformed depositor obtains by misrepresenting in states 1 and 2 and the right side is the expected utility of not misrepresenting in those states.

As individuals cannot distinguish between the two states they would assign a conditional probability $\pi_{1(1,2)}$ to receiving the highest second period consumption (i.e. when $\tilde{R} = R_h$ occurs) and $\pi_{2(1,2)}$ to receiving the lowest second period one (i.e. when $\tilde{R} = R_l$ occurs).

Similarly, the left side of Equation (10) is the utility that an uninformed depositor obtains by misrepresenting in state 5 and the right side is the utility of not misrepresenting in that state.

On the contrary, the uninformed agent will misrepresent in states $\theta = 3, 4$ and 6 ($\tilde{x}_{1U} = c_{11}$ and $\tilde{x}_{2U} = \tilde{c}_{21}$) provided the following condition holds:

$$\begin{aligned} & \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left\{ r \left[\frac{(k+c_{21}^{t_1})^{1-\gamma}}{1-\gamma} \pi_{1(3,4,6)} + \frac{(k+c_{21}^{t_1} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \pi_{2(3,4,6)} \right] + \right. \\ & (1-r) \left[\frac{(k+c_{21}^{t_2})^{1-\gamma}}{1-\gamma} \pi_{1(3,4,6)} + \frac{(k+c_{21}^{t_2} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \pi_{2(3,4,6)} \right] \Big\} \geq \\ & \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) \left\{ r \left[\frac{(k+c_{22}^{t_1})^{1-\gamma}}{1-\gamma} \pi_{1(3,4,6)} + \frac{(k+c_{22}^{t_1} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \pi_{2(3,4,6)} \right] + \right. \\ & (1-r) \left[\frac{(k+c_{22}^{t_2})^{1-\gamma}}{1-\gamma} \pi_{1(3,4,6)} + \frac{(k+c_{22}^{t_2} \frac{R_l}{R_h})^{1-\gamma}}{1-\gamma} \pi_{2(3,4,6)} \right] \Big\} \end{aligned} \quad (11)$$

$$\text{and } \pi_{1(3,4,6)} = \frac{(1-p)(1-r)(1-q)}{(1-r)[(1-q)+pq]+rpq}, \pi_{2(3,4,6)} = \frac{p[q+(1-r)(1-q)]}{(1-r)[(1-q)+pq]+rpq}.$$

The left side of Equation (11) is the expected utility that an uninformed depositor obtains by misrepresenting in states 3, 4 and 6 and the right side is the expected utility of not misrepresenting in those states.

Similarly, as individuals cannot distinguish among those states they would assign a conditional probability $\pi_{1(3,4,6)}$ to receiving the highest second period consumption and $\pi_{2(3,4,6)}$ to receiving the lowest second period one.

Having defined the optimal withdrawal decisions for all type of depositors at $T = 1$, conditions for both information-induced and pure panic runs to occur can then be summarized in theorem 1.

THEOREM 1. Assuming the condition given by Equation (2), Lemma 1 and Equations (9), (10), and (11), are satisfied, then there exists in the model an equilibrium with bank runs.¹⁰

¹⁰ It should be mentioned that although we have characterized one equilibrium of this model, in order to carry out welfare comparisons between suspension of convertibility and deposit insurance, there might exist other equilibria in this model.

Information-induced runs take place in states 3 and 6. The negative signal received by informed depositors induces the uninformed to withdraw too. However in state 4 there is a pure panic run as there is no adverse information held by any agent in that state. In this case, the uninformed have erroneously withdrawn their funds from the bank.

As mentioned above, whenever there are runs the bank **suspends convertibility** at the level of the highest proportion of type 1 consumers. It is assumed randomized arrivals (first) among only type 1 and pessimistic informed type 2 agents and finally any other panickers that will be frozen out of any early withdrawal given the suspension rule. The bank distributes the type 1 contract (between type 1s and informed type 2s) until a fraction equal to the highest proportion of type 1 consumers has withdrawn, after that, the bank only distributes the type 2 contract. This means that total withdrawals at date 1 will be:

$$t_2 c_{11} + (1 - t_2) c_{12} = K^{t_2}. \quad (12)$$

Let $\tilde{\beta}$ represent the random proportion of agents (type 1 and informed type 2) that are being rationed by the bank, that is, $\tilde{\beta}$ is such that:

$$\tilde{\beta} = \frac{\Delta \tilde{C} T_1}{\tilde{t}(c_{11} - c_{12}) + (1 - \tilde{t})\tilde{\alpha}(x_{1I} - c_{12})} \quad (13)$$

where:

$$\begin{aligned} \Delta \tilde{C} T_1 &= \tilde{C} T_1 - (1 - \tilde{t})(1 - \tilde{\alpha})(x_{1U} - c_{12}) - K^{t_2} & \text{if } \tilde{C} T_1 > K^{t_2} \\ \Delta \tilde{C} T_1 &= 0 & \text{if } \tilde{C} T_1 \leq K^{t_2} \end{aligned} \quad (14)$$

The ex ante expected utility with suspension, EU_{sus} , is:

$$\begin{aligned} EU_{sus} &= E_{\tilde{R}, \tilde{t}, \tilde{\alpha}} \left\{ \left[\rho_1 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1 - \rho_1) \frac{(k+\tilde{c}_{21})^{1-\gamma}}{1-\gamma} \right] \tilde{t}(1 - \tilde{\beta}) + \right. \\ &\quad \left[\rho_1 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho_1) \frac{(k+\tilde{c}_{22})^{1-\gamma}}{1-\gamma} \right] \tilde{t}\tilde{\beta} + \\ &\quad \left[\rho_2 \frac{(k+\tilde{x}_{1I})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \frac{(k+\tilde{x}_{2I})^{1-\gamma}}{1-\gamma} \right] (1 - \tilde{t})\tilde{\alpha}(1 - \tilde{\beta}) + \\ &\quad \left[\rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \frac{(k+\tilde{c}_{22})^{1-\gamma}}{1-\gamma} \right] (1 - \tilde{t})\tilde{\alpha}\tilde{\beta} + \\ &\quad \left. \left[\rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho_2) \frac{(k+\tilde{c}_{22})^{1-\gamma}}{1-\gamma} \right] (1 - \tilde{t})(1 - \tilde{\alpha}) \right\} \end{aligned} \quad (15)$$

and the welfare measure to be considered in this study is:

$$C_{sus} = \text{cert.equivalent}(EU_{sus}). \quad (16)$$

3.2. DEPOSIT INSURANCE

By contrast, let us now assume that there exists government **deposit insurance** of the bank deposits at $T = 2$. This insurance guarantees that whenever the bank fails (i.e., in those states of nature in which the low return is realized) individuals will be covered by the insurance fund. This means that an agent of type i will always receive the maximum second period consumption, $c_{2i}^{t_1}$ (in states 1, 2 and 3) or $c_{2i}^{t_2}$ (in states 4, 5 and 6), as if $R = R_h$ was realized. This insurance removes the incentives of informed individuals to act upon their information and hence to run on the bank as they are always assured the maximum second period return. Similarly, uninformed individuals will never have an incentive to withdraw. As a result, with a deposit insurance system bank runs will no longer occur and individuals will consume what was planned in the ex ante contract. Only type 1 depositors, who face liquidity needs, will withdraw their funds. The expected utility of individuals, when there is deposit insurance would be:

$$EU_{R_h} = \{r [t_1 U^1(c_{11}, c_{21}^{t_1}, \rho_1) + (1 - t_1) U^2(c_{12}, c_{22}^{t_1}, \rho_2)] + (1 - r) [t_2 U^1(c_{11}, c_{21}^{t_2}, \rho_1) + (1 - t_2) U^2(c_{12}, c_{22}^{t_2}, \rho_2)]\}. \quad (17)$$

Formally, the cost of the insurance could be expressed as follows: Whenever the realized benefit at date 2, i.e. $\tilde{R}(1 - K^{t_i})$ ¹¹ is less than the promised second period consumption, $t_i c_{21}^{t_i} + (1 - t_i) c_{22}^{t_i}$ ($i=1$ in states 1, 2 and 3 and $i=2$ in states 4, 5 and 6), then the difference will be supplied by the government, that is:¹²

$$\begin{aligned} \tilde{C}d &= 0 & \text{if} & \quad \tilde{R} = R_h \\ \tilde{C}d &= t_i c_{21}^{t_i} + (1 - t_i) c_{22}^{t_i} - \tilde{R}(1 - K^{t_i}) & \text{otherwise} \end{aligned} \quad (18)$$

The expected cost of the insurance is expressed as follows:

$$ECd = p \{r [t_1 c_{21}^{t_1} + (1 - t_1) c_{22}^{t_1} - R_l(1 - K^{t_1})] + (1 - r) [t_2 c_{21}^{t_2} + (1 - t_2) c_{22}^{t_2} - R_l(1 - K^{t_2})]\}. \quad (19)$$

Using the Laffont and Tirole (1986) approach¹³ we consider that a transfer Cd from the regulatory agent to the bank will generate a social cost of $(1 + \lambda)Cd$ and so the welfare measure for deposit insurance will be the certainty equivalent of the expected utility achieved with a demand deposit contract when individuals are assured the maximum second period return (C_{R_h}) ¹⁴ minus the social cost of the expected transfer from the public to the bank, $(1 + \lambda)ECd$, that is:

$$C_{dep} = C_{R_h} - (1 + \lambda)ECd \quad (20)$$

¹¹ K^{t_i} is the amount to be liquidated at date 1, contingent on the realization of \tilde{r} .

¹² $Cd > 0$ in 1, 3, 4 and 6. In states 1 and 4 the cost will be incurred with probability p .

¹³ See also Freixas and Gabilon (1999).

¹⁴ The expected utility is given in Equation (17).

Table II. Numerical data

t_1	t_2	r	α	q	p	R_l	R_h	\bar{R}	σ^2	ρ_1	ρ_2	γ	k
0.30	0.51	0.99	0.30	0.99	0.10	0.09	1.65	1.50	0.22	0.60	0.40	4.00	1

Table III. Suspension of convertibility

θ_i	State	Prob.	K^i	$\tilde{C}T_1$	Bank runs	$\tilde{\beta}$	Aggregate consumption (suspension)
i	$\tilde{i} \tilde{\alpha} \tilde{R}$	$p(\theta_i)$					
1	$t_1 0 \tilde{R}$	$r(1 - q)$	0.528	0.528	No	0	0.528
2	$t_1 \alpha R_h$	$r(1 - p)q$	0.528	0.528	No	0	0.528
3	$t_1 \alpha R_l$	rpq	0.528	0.638	Yes	> 0	0.561
4	$t_2 0 \tilde{R}$	$(1 - r)(1 - q)$	0.561	0.638	Yes	> 0	0.561
5	$t_2 \alpha R_h$	$(1 - r)(1 - p)q$	0.561	0.561	No	0	0.561
6	$t_2 \alpha R_l$	$(1 - r)pq$	0.561	0.638	Yes	> 0	0.561

Given the exogenous value of the deadweight tax, it is interesting to determine the value of λ^* for which the two contracts (suspension of convertibility and deposit insurance) deliver the same utility. This means λ^* should be such that $C_{\text{sus}} = C_{\text{dep}}$ or equivalently, $\lambda^* = \frac{C_{R_h} - C_{\text{sus}}}{ECd} - 1$. For the parameter values shown in Table II we would obtain $\lambda^* = 0.14$. This implies that if the cost of outside insurance is 14 per cent or lower, the deposit insurance arrangement is best. Otherwise, the decentralized contract is preferred.

Finally, Tables III and IV summarize the comparison between the two policies, suspension versus deposit insurance for each of the six possible states of nature and for the parameter values considered in Table II. The solution to the bank's problem (demand deposit contract) for this example would be: $c_{11} = 0.638$, $c_{21}^1 = 0.652$, $c_{21}^2 = 0.637$, $c_{12} = 0.481$, $c_{22}^1 = 0.837$, $c_{22}^2 = 0.820$.

Considering first the suspension measure (Table III), it can be observed that in states 1, 2 and 5 aggregate demand for liquidity ($\tilde{C}T_1$) coincides with the ex ante one (K^i). In these states there are no bank runs and as a result the proportion of agents rationed is zero ($\beta = 0$). In states 3, 4, and 6 there are total bank runs, as all agents demand the type 1 contract. In all of these states some agents will be rationed, ($\beta > 0$) and aggregate consumption will be equal to K^2 . In the case in which there is a deposit insurance system (Table IV) aggregate demand for liquidity coincides with the ex ante one and so there are never bank runs. However, avoiding runs by ensuring the highest second period outcome involves a cost in states 1, 3, 4 and 6 (although in states 1 and 4 the cost is only incurred with probability p).

Table IV. Deposit insurance

θ_i i	State $\tilde{t} \ \tilde{\alpha} \ \tilde{R}$	Prob. $p(\theta_i)$	$K^{\tilde{t}}$	$\tilde{C}T_1$	Bank runs	Cd	Aggregate consumption (insurance)
1	$t_1 \ 0 \ \tilde{R}$	$r(1 - q)$	0.528	0.528	No	0.0738*	0.528
2	$t_1 \ \alpha \ R_h$	$r(1 - p)q$	0.528	0.528	No	0	0.528
3	$t_1 \ \alpha \ R_l$	rpq	0.528	0.528	No	0.738	0.528
4	$t_2 \ 0 \ \tilde{R}$	$(1 - r)(1 - q)$	0.561	0.561	No	0.0686**	0.561
5	$t_2 \ \alpha \ R_h$	$(1 - r)(1 - p)q$	0.561	0.561	No	0	0.561
6	$t_2 \ \alpha \ R_l$	$(1 - r)pq$	0.561	0.561	No	0.686	0.561

$$*p[t_1 c_{21}^{t_1} + (1 - t_1)c_{22}^{t_1} - R_l(1 - K^{t_1})] = 0.10 * 0.738.$$

$$**p[t_2 c_{21}^{t_2} + (1 - t_2)c_{22}^{t_2} - R_l(1 - K^{t_2})] = 0.10 * 0.686.$$

4. Welfare comparisons: numerical examples

In this section an extensive set of numerical simulations has been carried out in order to determine the variation of the critical deadweight tax (λ^*) as a function of the relative risk aversion coefficient. This critical λ^* has been defined as the one for which the two contracts – suspension and deposit insurance – deliver the same utility. In all the numerical simulations, it has been assumed that the lowest proportion of type 1 agents (t_1) is realized with a high probability ($r = 0.99$). The motivation for this assumption is to create confusion between a large withdrawal queue size at the bank, due to a high liquidity shock or a negative information one. It is also assumed that the probability of the low value of the random return occurring is sufficiently small ($p = 0.10$), and this in turn simplifies the demand deposit contract (see Section 5 for an explanation). Finally, in all the numerical examples the conditions imposed by theorem 1 are always satisfied.

Let $t_1 = 0.30$, $r = 0.99$, $\alpha = 0.30$, $q = 0.99$, $p = 0.10$, $\bar{R} = 1.50$, $\sigma^2 = 0.22$, $\rho = 0.60$, $k = 1$.¹⁵ Given these parameters, the figures represent the critical value of the deadweight tax as a function of the relative risk aversion coefficient, that is, $\lambda^* = \lambda^*(\gamma)$. In Figure 2 this function is represented for three different values of the variance in the long term asset return (σ^2), i.e R_l and R_h are chosen so that \bar{R} and the rest of the parameters remain constant. Similarly in Figure 3, the function $\lambda^* = \lambda^*(\gamma)$ is shown for three values of the expected return (\bar{R}). Finally, Figures 4 and 5 represent the function for three different values of the intertemporal discount factor (ρ) and the probability of having informed agents (q). It can be observed that all Figures follow the general pattern represented in Figure 1: If individuals are not very risk averse or more precisely, for values of γ below a critical one γ_c , the decentralized contract is always best, even though in these cases deposit insurance is available at zero cost. However, as risk aversion increases, the

¹⁵ $t_2 = t_1 + \alpha(1 - t_1) = 0.51$.

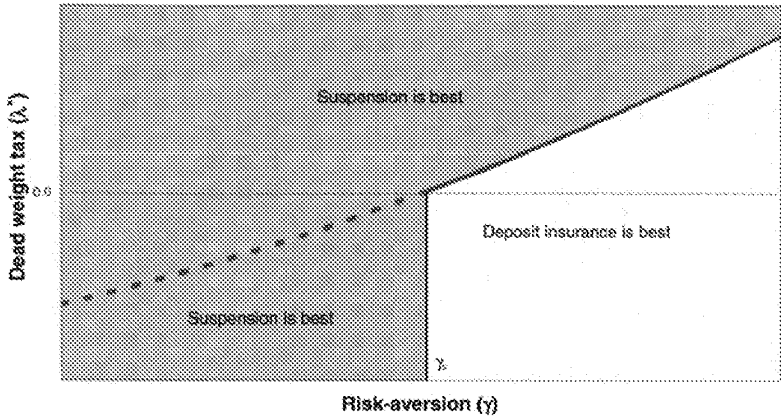


Figure 1. Deadweight tax (λ^*) for which the two contracts deliver the same utility as a function of the relative risk aversion coefficient. General pattern.

cutoff value of the deadweight tax (λ^*) also increases and so the deposit insurance arrangement is best for a larger set of parameters. As mentioned before, if the value of the deadweight tax is λ^* per cent or lower, the deposit insurance arrangement is best. Otherwise, the decentralized contract is preferred.

As Figures 2–5 indicate, the value of λ^* for which the two contracts deliver the same utility is influenced by other parameters of the model: increasing the dispersion in the long term asset return or decreasing its expected return (Figures 2 and 3) increase the value of λ^* (and this effect also increases with risk aversion). The decentralized contract performs better when the bank's investment is of low risk and high expected return. Figure 4 shows that increasing the agents' intertemporal discount factor also increases this value of λ^* . A higher value of ρ worsens the suspension measure by imposing a higher liquidity cost on type 1 consumers. Finally, the probability of having informed agents (Figure 5) does not influence the value of λ^* .

From the above examples it seems that if individuals are not very risk averse then the suspension contract is the best way to allocate resources even though in these cases deposit insurance is available at zero cost. We could conclude that in general information-based bank runs may be good in an ex ante welfare sense: Given that the deposit contract does not allow first period allocations to depend on the signal, information-based runs may be beneficial in the sense that they do allow first period consumption outcomes to depend on the aggregate shock (although in an indirect and limited way). On the contrary, a deposit insurance system, which eliminates information-induced runs does not, and therefore any policy which tries to eliminate runs has a negative side to it. However, as already pointed out in the paper this result depends on the value of the deadweight tax (λ). The above figures represented the value of λ^* for which the two contracts deliver the same utility. As

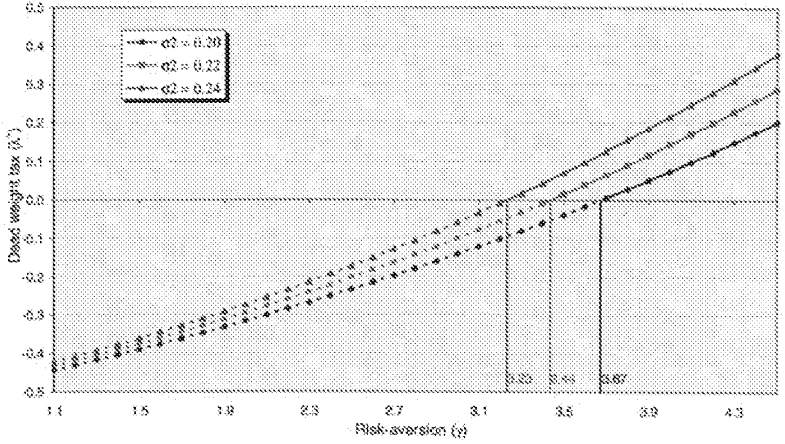


Figure 2. Variation in the dispersion in the long term asset return (σ^2).

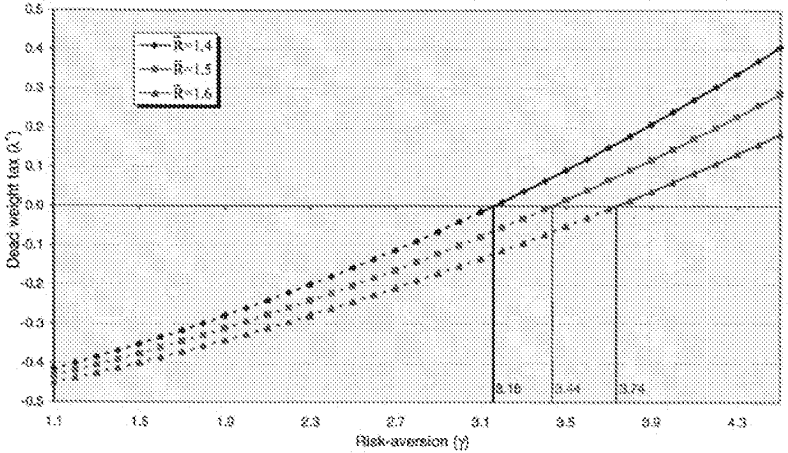


Figure 3. Variation in the expected return (\bar{R}).

the comparative statics results indicate this value is increasing in the level of risk aversion, the dispersion in the long-term asset return or the intertemporal discount factor coefficient. On the contrary, it is decreasing in the expected return.

5. Discussion of the assumptions

This section provides a discussion of some of the assumptions that have been introduced in the paper to model the contracts:

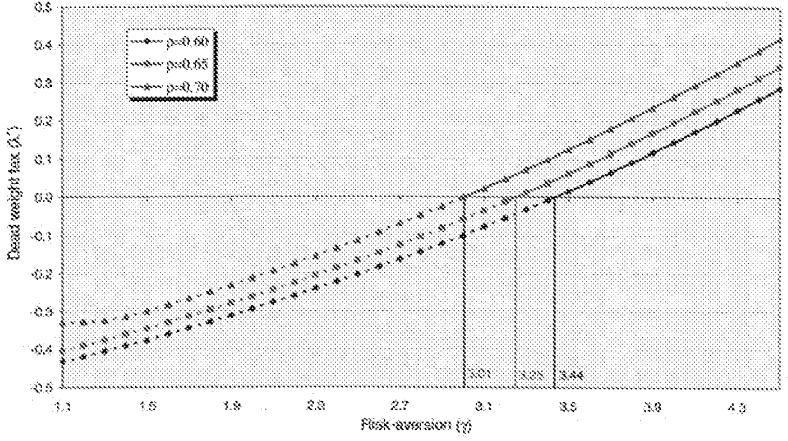


Figure 4. Variation in the intertemporal discount factor (ρ).

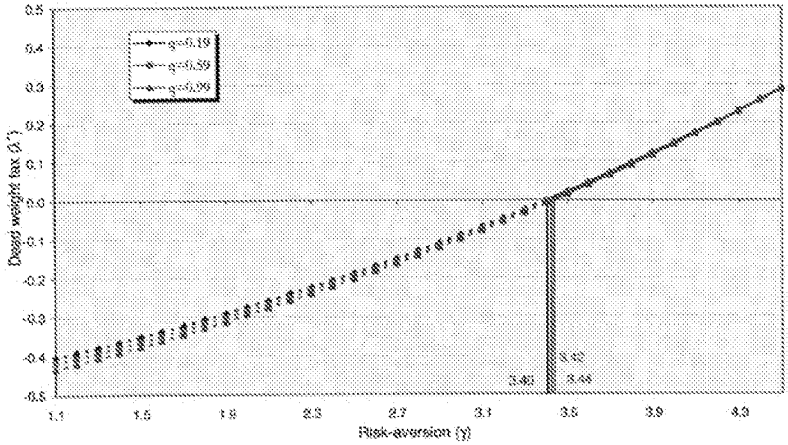


Figure 5. Variation in the information structure (q).

(1) The deposit contract (Section 3): It has been assumed that the probability of the low return occurring is sufficiently small and this allows to simplify the demand deposit contract. The contract is designed ignoring the information shock at $T = 1$ and so bank runs will occur under certain conditions. It is shown elsewhere (see Samartin (2002)) that if the probability of the low return is sufficiently small a contract that allows for runs would dominate a contract that is designed so that runs are always prevented (this last contract would have two additional incentive constraints that describe when it is rational for an informed agent to truthfully reveal his or her type). Also the former contract approximates the complete (optimal) contract.

The intuition for the result is that in order to change the deposit contract so that depositors have an incentive not to run, their payoffs have to be altered in all states of nature, hence, a significant loss is incurred with high probability and the gain is only realized with low probability. Finally, it should be mentioned that other papers in this banking literature (e.g., Jacklin and Bhattacharya (1988), Alonso (1996)) also consider deposit contracts along the same lines.

(2) Timing of arrivals (Subsection 3.1). As in Chari and Jagannathan (1988), it is assumed that type 1 and informed type 2 make their withdrawal decision first, followed by other uninformed type 2 who might panic if the “first stage queue” is large enough. This assumption (together with the fact that there is suspension after the highest proportion of type 1 agents has withdrawn) avoids multiple equilibria (of the Diamond and Dybvig’s type), that might otherwise occur if all agents made their withdrawal decisions at the same time and the long term technology can be liquidated prematurely. In this sense, the paper focuses on one equilibrium that occurs under certain parameters and analyses its welfare properties.

(3) Deposit insurance (Subsection 3.2): Finally, it is assumed that there exists a deposit insurance system that always guarantees depositors their promised payment (the maximum second period consumption). This implies that whenever the bank fails (i.e. in those states of nature in which the low return is realized), it receives a transfer from the regulatory agent to pay out individuals. The cost of deposit insurance is that when asset returns are low other sectors have to be taxed to make up the shortfall. Using the Laffont and Tirole (1986) approach it is considered that a transfer Cd from the regulatory agent to the bank will generate a social cost of $(1 + \lambda)Cd$. This cost (the value of the deadweight tax) is exogenous. Comparative statics results have been presented by looking at the value of λ^* for which the two contracts deliver the same utility. However, a possible extension of the model could be to consider an endogenous value of this deadweight tax.

(4) For simplicity, liquidation costs have been neglected. However, an adequate model of interim liquidation values for the long term asset should be of interest.

6. Concluding Remarks

This paper introduces risk-averse preferences in Chari and Jagannathan’s model. A first motivation for this extension is to give a positive role for a financial intermediary in the economy, that offers risk sharing contracts to liquidity seeking individuals. In this context, the demand deposit contract is derived and both information-induced and pure panic runs will occur in some states of the world. A second, and more important, motivation for this extension is to complete Chari and Jagannathan’s welfare analysis, by comparing two second based mechanisms for dealing with runs, namely suspension of convertibility or deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs

or deadweight taxation).¹⁶ With the first mechanism, payments are suspended at a certain level and with the second one, deposits are always guaranteed when the bank fails.

The numerical examples represented the value of the deadweight tax (λ^*) for which the two contracts deliver the same utility, as a function of the relative risk aversion coefficient. A general feature to these graphs is that if individuals are not very risk averse then the decentralized contract is always best, even though deposit insurance is available at zero cost. It is also shown that this value of λ^* is increasing in the level of risk aversion in the population, in the dispersion in the long-term asset return and in the intertemporal discount factor coefficient. On the contrary, it is decreasing in the expected return.

Appendix A. The Demand Deposit Contract

The problem to be solved is:

$$\max_{c_{ij}, K} \left\{ E_{\tilde{R}, \tilde{t}} [\tilde{t} U^1(c_{11}, \tilde{c}_{21}, \rho_1) + (1 - \tilde{t}) U^2(c_{12}, \tilde{c}_{22}, \rho_2)] \right\}, \quad (21)$$

$$\begin{aligned} \text{s.t. } t_1(c_{11} + \frac{c_{21}^{t_1}}{R_h}) + (1 - t_1)(c_{12} + \frac{c_{22}^{t_1}}{R_h}) &= 1 \\ t_2(c_{11} + \frac{c_{21}^{t_2}}{R_h}) + (1 - t_2)(c_{12} + \frac{c_{22}^{t_2}}{R_h}) &= 1 \end{aligned} \quad (22)$$

$$E_{\tilde{R}, \tilde{t}} [U^j(c_{1j}, \tilde{c}_{2j}, \rho_j)] \geq E_{\tilde{R}, \tilde{t}} [U^j(c_{1i}, \tilde{c}_{2i}, \rho_j)] \text{ for } i \neq j; i, j = 1, 2. \quad (23)$$

The FOCS are the following ones:

$$\frac{\partial L}{\partial c_{11}} = (k + c_{11})^{-\gamma} t \rho - t_1 \lambda_1 - t_2 \lambda_2 - (1 - \rho) \lambda_3 (k + c_{11})^{-\gamma} + \rho \lambda_4 (k + c_{11})^{-\gamma} = 0 \quad [a]$$

$$\begin{aligned} \frac{\partial L}{\partial c_{21}^{t_1}} &= \left[(k + c_{21}^{t_1})^{-\gamma} (1 - p) + p (k + c_{21}^{t_1} R_l / R_h)^{-\gamma} \frac{R_l}{R_h} \right] r \\ &\times [t_1 (1 - \rho) - \rho \lambda_3 + (1 - \rho) \lambda_4] - t_1 \frac{1}{R_h} \lambda_1 = 0 \end{aligned} \quad [b]$$

$$\begin{aligned} \frac{\partial L}{\partial c_{12}} &= (k + c_{12})^{-\gamma} (1 - t) (1 - \rho) - (1 - t_1) \lambda_1 - (1 - t_2) \lambda_2 \\ &+ \lambda_3 (1 - \rho) (k + c_{12})^{-\gamma} - \lambda_4 (k + c_{12})^{-\gamma} = 0 \end{aligned} \quad [c]$$

$$\begin{aligned} \frac{\partial L}{\partial c_{22}^{t_2}} &= \left[(k + c_{22}^{t_2})^{-\gamma} (1 - p) + p (k + c_{22}^{t_2} R_l / R_h)^{-\gamma} \frac{R_l}{R_h} \right] r \\ &\times [(1 - t_1) \rho + \rho \lambda_3 - (1 - \rho) \lambda_4] - (1 - t_1) \frac{1}{R_h} \lambda_1 = 0 \end{aligned} \quad [d]$$

$$\begin{aligned} \frac{\partial L}{\partial c_{21}^{t_2}} &= \left[(k + c_{21}^{t_2})^{-\gamma} (1 - p) + p (k + c_{21}^{t_2} R_l / R_h)^{-\gamma} \frac{R_l}{R_h} \right] (1 - r) \\ &\times [t_2 (1 - \rho) - \rho \lambda_3 + (1 - \rho) \lambda_4] - t_2 \frac{1}{R_h} \lambda_2 = 0 \end{aligned} \quad [e]$$

¹⁶ These mechanisms may be interpretable as government or market-institution-based policies. Some authors (e.g., Gorton (1985), Villamil (1991), Selgin and White (1997)) have argued that the use of suspension by banks was a way to rule out runs without any need for state intervention.

$$\begin{aligned}
\frac{\partial L}{\partial c_{22}^{t_2}} &= \left[(k + c_{22}^{t_2})^{-\gamma} (1 - p) + p(k + c_{22}^{t_2} R_l / R_h)^{-\gamma} \frac{R_l}{R_h} \right] r_2 \\
&\quad \times [(1 - t_2)\rho + \rho\lambda_3 - (1 - \rho)\lambda_4] - (1 - t_2) \frac{1}{R_h} \lambda_2 = 0 \quad [f] \\
\frac{\partial L}{\partial \lambda_1} &= 1 - t_1 \left(c_{11} + \frac{c_{21}^{t_1}}{R_h} \right) - (1 - t_1) \left(c_{12} + \frac{c_{22}^{t_1}}{R_h} \right) = 0 \quad [g] \\
\frac{\partial L}{\partial \lambda_2} &= 1 - t_2 \left(c_{11} + \frac{c_{21}^{t_2}}{R_h} \right) - (1 - t_2) \left(c_{12} + \frac{c_{22}^{t_2}}{R_h} \right) = 0 \quad [h] \\
\frac{\partial L}{\partial \lambda_3} &= (1 - \rho) \frac{(k + c_{12})^{1-\gamma}}{1-\gamma} + \rho \left\{ r \left[\frac{(k + c_{22}^{t_1})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{22}^{t_1} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right. \\
&\quad \left. + r_2 \left[\frac{(k + c_{22}^{t_2})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{22}^{t_2} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right\} \\
&\quad - (1 - \rho) \frac{(k + c_{11})^{1-\gamma}}{1-\gamma} + \rho \left\{ r \left[\frac{(k + c_{21}^{t_1})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{21}^{t_1} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right. \\
&\quad \left. + r_2 \left[\frac{(k + c_{21}^{t_2})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{21}^{t_2} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right\} \geq 0 \quad [i] \\
\frac{\partial L}{\partial \lambda_4} &= \rho \frac{(k + c_{11})^{1-\gamma}}{1-\gamma} + (1 - \rho) \left\{ r \left[\frac{(k + c_{21}^{t_1})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{21}^{t_1} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right. \\
&\quad \left. + (1 - r) \left[\frac{(k + c_{21}^{t_2})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{21}^{t_2} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right\} \\
&\quad - \rho \frac{(k + c_{12})^{1-\gamma}}{1-\gamma} + (1 - \rho) \left\{ r \left[\frac{(k + c_{22}^{t_1})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{22}^{t_1} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right. \\
&\quad \left. + (1 - r) \left[\frac{(k + c_{22}^{t_2})^{1-\gamma}}{1-\gamma} (1 - p) + \frac{(k + c_{22}^{t_2} R_l / R_h)^{1-\gamma}}{1-\gamma} p \right] \right\} \geq 0 \quad [j]
\end{aligned} \tag{24}$$

This system of non linear equations has been solved applying the Newton Raphson technique.

In the particular case in which $R_l \simeq 0$, the FOCS would become:

$$\begin{aligned}
\frac{\partial L}{\partial c_{11}} &= \bar{c}_{11}^{-\gamma} t - t_1 \lambda_1 - t_2 \lambda_2 - \mu \lambda_3 \bar{c}_{11}^{-\gamma} + \lambda_4 \bar{c}_{11}^{-\gamma} = 0 \quad [a] \\
\frac{\partial L}{\partial c_{21}^{t_1}} &= \bar{c}_{21}^{t_1 - \gamma} \mu (1 - p) r - \frac{1}{R_h} \lambda_1 - \lambda_3 \bar{c}_{21}^{t_1 - \gamma} (1 - p) r / t_1 \\
&\quad + \lambda_4 \bar{c}_{21}^{t_1 - \gamma} \mu r (1 - p) / t_1 = 0 \quad [b] \\
\frac{\partial L}{\partial c_{12}} &= \bar{c}_{12}^{-\gamma} (1 - t) \mu - (1 - t_1) \lambda_1 - (1 - t_2) \lambda_2 + \lambda_3 \mu \bar{c}_{12}^{-\gamma} - \lambda_4 \bar{c}_{12}^{-\gamma} = 0 \quad [c] \\
\frac{\partial L}{\partial c_{22}^{t_1}} &= \bar{c}_{22}^{t_1 - \gamma} (1 - p) r - \frac{1}{R_h} \lambda_1 + \bar{c}_{22}^{t_1 - \gamma} \lambda_3 (1 - p) \frac{r}{1 - t_1} \\
&\quad - \lambda_4 \bar{c}_{22}^{t_1 - \gamma} \mu (1 - p) \frac{r}{1 - t_1} = 0 \quad [d] \\
\frac{\partial L}{\partial c_{21}^{t_2}} &= \bar{c}_{21}^{t_2 - \gamma} \mu (1 - p) (1 - r) - \frac{1}{R_h} \lambda_2 - \lambda_3 \bar{c}_{21}^{t_2 - \gamma} (1 - p) \frac{(1 - r)}{t_2} \\
&\quad + \lambda_4 \bar{c}_{21}^{t_2 - \gamma} \mu (1 - r) \frac{1 - p}{t_2} = 0 \quad [e] \\
\frac{\partial L}{\partial c_{22}^{t_2}} &= \bar{c}_{22}^{t_2 - \gamma} (1 - p) (1 - r) - \frac{1}{R_h} \lambda_2 + \bar{c}_{22}^{t_2 - \gamma} \lambda_3 (1 - p) \frac{(1 - r)}{1 - t_2} \\
&\quad - \bar{c}_{22}^{t_2 - \gamma} \lambda_4 \mu (1 - p) \frac{(1 - r)}{1 - t_2} = 0 \quad [f]
\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda_1} &= 1 + \left(1 + \frac{1}{R_h}\right)k - t_1 \left(\bar{c}_{11} + \frac{\bar{c}_{21}^{t_1}}{R_h}\right) - (1 - t_1) \left(\bar{c}_{12} + \frac{\bar{c}_{22}^{t_1}}{R_h}\right) = 0 \quad [g] \\ \frac{\partial L}{\partial \lambda_2} &= 1 + \left(1 + \frac{1}{R_h}\right)k - t_2 \left(\bar{c}_{11} + \frac{\bar{c}_{21}^{t_2}}{R_h}\right) - (1 - t_2) \left(\bar{c}_{12} + \frac{\bar{c}_{22}^{t_2}}{R_h}\right) = 0 \quad [h]\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda_3} &= \mu \frac{\bar{c}_{12}^{1-\gamma}}{1-\gamma} + (1-p) \left[\frac{\bar{c}_{22}^{t_1} 1-\gamma}{1-\gamma} r + \frac{\bar{c}_{22}^{t_2} 1-\gamma}{1-\gamma} (1-r) \right] \\ &\quad - \mu \frac{\bar{c}_{11}^{1-\gamma}}{1-\gamma} - (1-p) \left[\frac{\bar{c}_{21}^{t_1} 1-\gamma}{1-\gamma} r + \frac{\bar{c}_{21}^{t_2} 1-\gamma}{1-\gamma} (1-r) \right] \geq 0 \quad [i] \\ \frac{\partial L}{\partial \lambda_4} &= \frac{\bar{c}_{11}^{1-\gamma}}{1-\gamma} + \mu(1-p) \left[\frac{\bar{c}_{21}^{t_1} 1-\gamma}{1-\gamma} r + \frac{\bar{c}_{21}^{t_2} 1-\gamma}{1-\gamma} (1-r) \right] \\ &\quad - \frac{\bar{c}_{12}^{1-\gamma}}{1-\gamma} - \mu(1-p) \left[\frac{\bar{c}_{22}^{t_1} 1-\gamma}{1-\gamma} r + \frac{\bar{c}_{22}^{t_2} 1-\gamma}{1-\gamma} (1-r) \right] \geq 0 \quad [j]\end{aligned} \quad (25)$$

where $\mu = \frac{1-\rho}{\rho}$ and $\bar{c}_{ij} = c_{ij} + k$.

(i) The two incentive constraints are never binding ($\lambda_3 = 0$, $\lambda_4 = 0$).

Let $t = rt_1 + (1-r)t_2$, $d_i = t_i \mu^{\frac{1}{\gamma}} + 1 - t_i$ ($i = 1, 2$). In this case the optimal solution will be:

$$\begin{aligned}\bar{c}_{11} &= A - \left[\frac{1-t_1}{\Delta t} d_1 \frac{\bar{c}_{22}^{t_1}}{R_h} - \frac{1-t_2}{\Delta t} d_2 \frac{\bar{c}_{22}^{t_2}}{R_h} \right] \\ \bar{c}_{12} &= A + \left[\frac{t_1}{\Delta t} d_1 \frac{\bar{c}_{22}^{t_1}}{R_h} - \frac{t_2}{\Delta t} d_2 \frac{\bar{c}_{22}^{t_2}}{R_h} \right] \\ \bar{c}_{21}^{t_i} &= \mu^{\frac{1}{\gamma}} \bar{c}_{22}^{t_i} \quad \text{and } i = 1, 2\end{aligned} \quad (26)$$

and $\bar{c}_{22}^{t_i}$ ($i = 1, 2$) are obtained from the following system of non-linear equations:

$$\begin{aligned}\mu \left[A + \frac{t_1}{\Delta t} d_1 \frac{\bar{c}_{22}^{t_1}}{R_h} - \frac{t_2}{\Delta t} d_2 \frac{\bar{c}_{22}^{t_2}}{R_h} \right]^{-\gamma} &= R_h(1-p) \left[r \frac{1-t_1}{1-t} \bar{c}_{22}^{t_1-\gamma} \right. \\ &\quad \left. + (1-r) \frac{1-t_2}{1-t} \bar{c}_{22}^{t_2-\gamma} \right]\end{aligned} \quad (27)$$

$$\left[A - \frac{1-t_1}{\Delta t} d_1 \frac{\bar{c}_{22}^{t_1}}{R_h} - \frac{1-t_2}{\Delta t} d_2 \frac{\bar{c}_{22}^{t_2}}{R_h} \right]^{-\gamma} = R_h(1-p) \left[r \frac{t_1}{t} \bar{c}_{22}^{t_1-\gamma} + (1-r) \frac{t_2}{t} \bar{c}_{22}^{t_2-\gamma} \right] \quad (28)$$

It can be easily shown that this case is satisfied if the following conditions are satisfied:

$$\left(1 - \mu^{\frac{1-\gamma}{\gamma}}\right) \left(r \frac{\bar{c}_{22}^{t_1} 1-\gamma}{1-\gamma} + (1-r) \frac{\bar{c}_{22}^{t_2} 1-\gamma}{1-\gamma} \right) \geq \frac{\mu}{1-p} \left(\frac{\bar{c}_{11}^{1-\gamma}}{1-\gamma} - \frac{\bar{c}_{12}^{1-\gamma}}{1-\gamma} \right) \quad (29)$$

$$\left(\mu^{\frac{1-\gamma}{\gamma}} - 1\right) \left(r \frac{\bar{c}_{22}^{t_1^{1-\gamma}}}{1-\gamma} + (1-r) \frac{\bar{c}_{22}^{t_2^{1-\gamma}}}{1-\gamma} \right) \geq \frac{1}{(1-p)\mu} \left(\frac{\bar{c}_{12}^{1-\gamma}}{1-\gamma} - \frac{\bar{c}_{11}^{1-\gamma}}{1-\gamma} \right) \quad (30)$$

(ii) The incentive constraint for type 1 agents is not binding and that of type 2 is binding ($\lambda_3 = 0$, $\lambda_4 > 0$).

The optimal solution in this case is defined by the following set of equations:

$$\bar{c}_{11} = A - \frac{1}{R_h} \left[\frac{1-t_1}{\Delta t} (t_1 \bar{c}_{21}^{t_1} + (1-t_1) \bar{c}_{22}^{t_1}) - \frac{1-t_2}{\Delta t} (t_2 \bar{c}_{21}^{t_2} + (1-t_2) \bar{c}_{22}^{t_2}) \right] \quad (31)$$

$$\bar{c}_{12} = A + \frac{1}{R_h} \left[\frac{t_1}{\Delta t} (t_1 \bar{c}_{21}^{t_1} + (1-t_1) \bar{c}_{22}^{t_1}) - \frac{t_2}{\Delta t} (t_2 \bar{c}_{21}^{t_2} + (1-t_2) \bar{c}_{22}^{t_2}) \right] \quad (32)$$

$$\mu \bar{c}_{12}^{1-\gamma} + (1-p) \left(r \bar{c}_{22}^{t_1^{1-\gamma}} + (1-r) \bar{c}_{22}^{t_2^{1-\gamma}} \right) = \mu \bar{c}_{11}^{1-\gamma} + (1-p) \left(r \bar{c}_{21}^{t_1^{1-\gamma}} + (1-r) \bar{c}_{21}^{t_2^{1-\gamma}} \right) \quad (33)$$

$$\begin{aligned} \left(\frac{1}{t_1} + \frac{\mu}{1-t_1} \right) (\bar{c}_{21}^{t_1} \bar{c}_{22}^{t_1})^{-\gamma} + A_2 \left(-\frac{1}{1-t_1} \bar{c}_{22}^{t_1^{-\gamma}} + \frac{1}{t_1} \bar{c}_{21}^{t_1^{-\gamma}} \right) \\ + B_2 \left(\mu \bar{c}_{21}^{t_1^{-\gamma}} - \bar{c}_{22}^{t_1^{-\gamma}} \right) = 0 \end{aligned} \quad (34)$$

$$\begin{aligned} \left(\frac{1}{t_2} + \frac{\mu}{1-t_2} \right) (\bar{c}_{21}^{t_2} \bar{c}_{22}^{t_2})^{-\gamma} + A_1 \left(-\frac{1}{1-t_2} \bar{c}_{22}^{t_2^{-\gamma}} + \frac{1}{t_2} \bar{c}_{21}^{t_2^{-\gamma}} \right) \\ + B_1 \left(\mu \bar{c}_{21}^{t_2^{-\gamma}} - \bar{c}_{22}^{t_2^{-\gamma}} \right) = 0 \end{aligned} \quad (35)$$

$$\frac{\mu \bar{c}_{21}^{t_1^{-\gamma}} - \bar{c}_{22}^{t_1^{-\gamma}}}{\frac{1}{t_1} \bar{c}_{21}^{t_1^{-\gamma}} + \frac{1}{1-t_1} \bar{c}_{22}^{t_1^{-\gamma}}} = \frac{-\mu \bar{c}_{21}^{t_2^{-\gamma}} - \bar{c}_{22}^{t_2^{-\gamma}}}{\frac{1}{t_2} \bar{c}_{21}^{t_2^{-\gamma}} + \frac{1}{1-t_2} \bar{c}_{22}^{t_2^{-\gamma}}} \quad (36)$$

where:

$$A_2 = \frac{(1-t) t_2 \mu \bar{c}_{12}^{-\gamma} - t(1-t_2) \bar{c}_{11}^{-\gamma}}{(1-p) r R_h \Delta t} \quad (37)$$

$$B_2 = \frac{t_2 \bar{c}_{12}^{-\gamma} + (1-t_2) \bar{c}_{11}^{-\gamma}}{(1-p) r R_h \Delta t} \quad (38)$$

$$A_1 = \frac{(1-t) t_1 \mu \bar{c}_{12}^{-\gamma} - t(1-t_1) \bar{c}_{11}^{-\gamma}}{(1-p)(1-r) R_h \Delta t} \quad (39)$$

$$B_1 = \frac{t_1 \bar{c}_{12}^{-\gamma} + (1 - t_1) \bar{c}_{11}^{-\gamma}}{(1 - p)(1 - r)R_h \Delta t} \quad (40)$$

The case $\lambda_3 > 0$, $\lambda_4 = 0$ is obtained in a similar way.

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